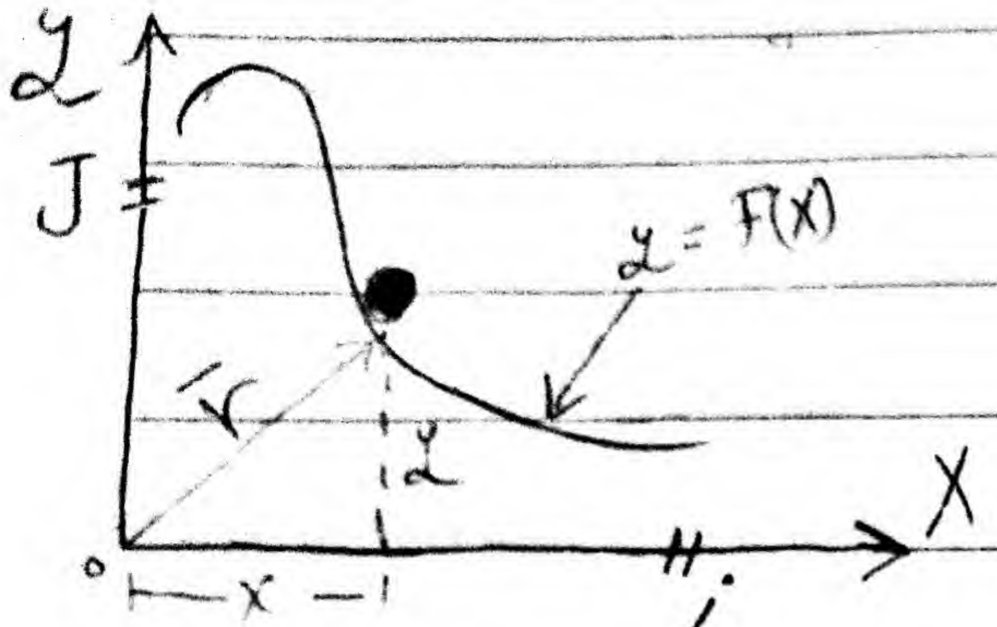
الباب الأول

الحركة في المستوى

١- الاحداثيات الكارتيزية :-



- الموضع $\bar{r}$:

$$\bar{r} = X\hat{i} + y\hat{J}$$

- المعادلات الزمنية :

$$X = F(t)$$

$$y = F(t)$$

- متجه سرعة المسيح :

$$\bar{V} = \frac{d\bar{r}}{dt} = \frac{dX}{dt}\hat{i} + \frac{dy}{dt}\hat{J}$$

$$\bar{V} = V_x\hat{i} + V_y\hat{j}$$

$$V = \sqrt{V_x^2 + V_y^2}$$

$$\tan\theta = \frac{V_y}{V_x}$$

- متجه العجلة $\bar{f}$:

$$\bar{f} = \frac{d\bar{V}}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j}$$

$$\bar{f} = F_x\hat{i} + F_y\hat{j}$$

$$f = \sqrt{F_x^2 + F_y^2}$$

$$\tan\phi = \frac{F_y}{F_x}$$

معادلة المسار

هي علاقة بين x و y بدون الزمن

EX (8)

Pg: 16

$\bar{V} = 2t\hat{i} + 3t^2\hat{j}$

$t = 0$

$\bar{r}_0 = 5\hat{i} - 8\hat{j}$

(a) Sol

$\bar{f} = \frac{d\bar{V}}{dt} = 2\hat{i} + 6t\hat{j}$

$f = \sqrt{4 + 36t^2}$ $t = 3$ Sec

$f = r_r$

$\theta = \tan^{-1}(3t) = \tan^{-1}(9) = r_r$

(ب)

$V = \frac{d\bar{r}}{dt} = 2t\hat{i} + 3t^2\hat{j}$

$\int d\bar{r} = \int ((2t)\hat{i} + (3t^2)\hat{j})\, dt$

$\bar{r} = t^2\hat{i} + t^3\hat{j} + \bar{C}$

$t = 0$, $r = 5\hat{i} - 8\hat{j}$

$C = 5\hat{i} - 8\hat{j}$

$\bar{r} = (t^2 + 5)\hat{i} + (t^3 - 8)\hat{j}$

$\bar{r}_{t=3} = 14\hat{i} + 19\hat{j}$

(c)

$x = t^2 + 5$ ①

$y = t^3 - 8$ ②

من ①

$t = (x - 5)^{\frac{1}{2}}$

$y = (x - 5)^{\frac{3}{2}} - 8$

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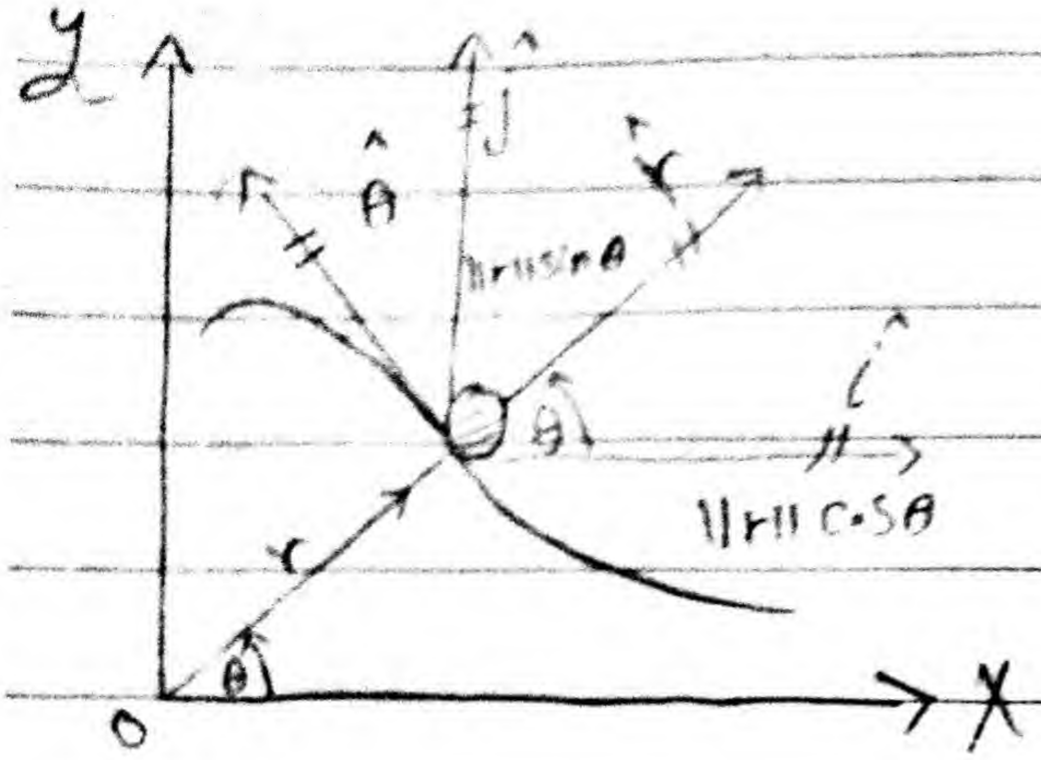
٢- الاحداثيات القطبية :-



الموضع :- $r = f(t)$

$\theta = f(t)$.

$$\bar{r} = r\hat{r}$$

$$\hat{r} = \cos\theta\, \hat{i} + \sin\theta\, \hat{j}$$

$$\hat{\theta} = -\sin\theta\, \hat{i} + \cos\theta\, \hat{j}$$

$$\frac{d\hat{\theta}}{dt} = -\cos\theta \frac{d\theta}{dt} \hat{i} + -\sin\theta \frac{d\theta}{dt} \hat{j}$$

$$= -\frac{d\theta}{dt}(\cos\theta\hat{i} + \sin\theta\hat{j}) = -\theta^{\circ}\hat{r}$$

السرعة :-

$$\bar{V} = \frac{d\bar{r}}{dt}$$

$$= \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\frac{d\hat{r}}{dt} = -\sin\theta \frac{d\theta}{dt} \hat{i} + \cos\theta \frac{d\theta}{dt} \hat{j}$$

$$= \frac{d\theta}{dt}(-\sin\theta\, \hat{i} + \cos\theta\, \hat{j})$$

$$\frac{d\hat{r}}{dt} = \theta^{\circ}\, \hat{\theta}$$

$$\bar{V} = \dot{r}\hat{r} + r\theta^{\circ}\hat{\theta}$$

$$= V_r \hat{r} + V_\theta \hat{\theta}$$

$$V_r = r^{\circ} \quad \& \quad V_\theta = r\theta^{\circ}$$

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$$\bar{F} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(r^{\circ}\hat{r} + r\theta^{\circ}\hat{\theta})$$

$$= r^{\circ\circ}\hat{r} + \theta^{\circ}\hat{\theta}r^{\circ} + r^{\circ}\theta^{\circ}\hat{\theta} + r\theta^{\circ\circ}\hat{\theta} + r\theta^{\circ}\frac{d\hat{\theta}}{dt}$$

$$= r^{\circ\circ}\hat{r} + 2r^{\circ}\theta^{\circ}\hat{\theta} + r\theta^{\circ\circ}\hat{\theta} - r\theta^{\circ 2}\hat{r}$$

$$\bar{f} = (r^{\circ\circ} - r\theta^{\circ 2})\hat{r} + (r\theta^{\circ\circ} + 2r^{\circ}\theta^{\circ})\hat{\theta}$$

$$f_r = r^{\circ\circ} - r\theta^{\circ 2}$$

$$f_\theta = r\theta^{\circ\circ} + 2r^{\circ}\theta^{\circ}$$

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EX (10)

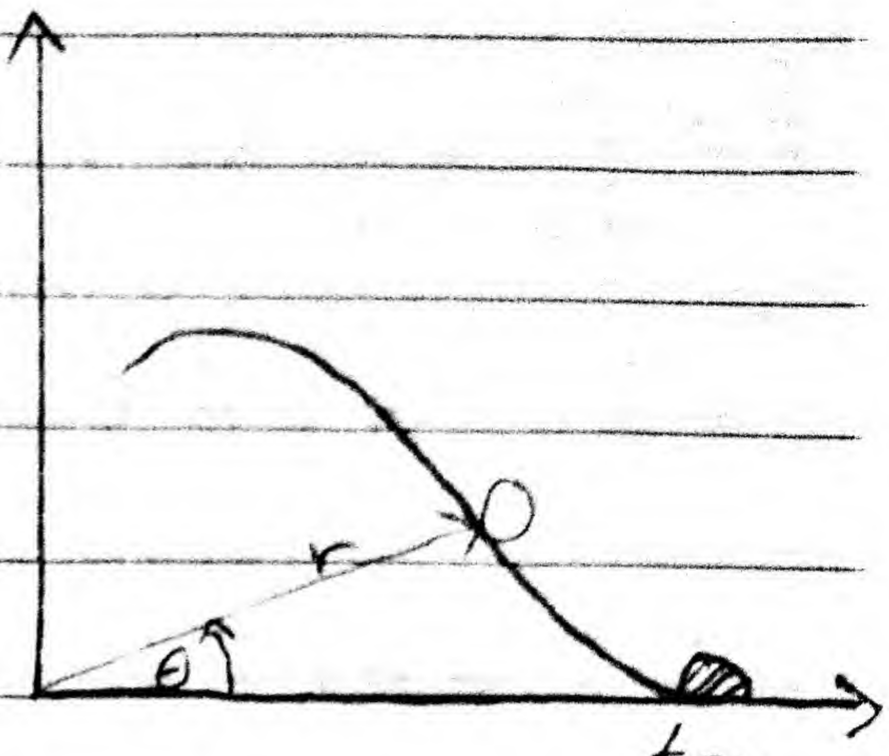
fig ; 21

$r = a + b\theta$

v & f = f(t)

t = 0

$\theta = 0$



Sol

$\omega = \frac{d\theta}{dt}$

$\int d\theta = \int \omega \, dt$

$\theta = \omega t + C$ $\quad t = 0, \theta = 0$

$\theta = \omega t$

$r = a + b\theta$, $\theta = \omega t$

$r = a + b\omega t$

$r^\circ = b\omega$ $\quad \theta^\circ = \omega$

$r^{\circ\circ} = 0$ $\quad \theta^{\circ\circ} = 0$

مركبات السرعة:-

$V_r = b\omega$

$V_\theta = \omega(a + b\omega t)$

$V = \sqrt{V_r^2 + V_\theta^2}$

$V_r = r^\circ$

$V_\theta = r\theta^\circ$

مركبات العجلة:-

$f_r = 0 - (a + b\omega t)\omega^2$

$f_\theta = 0 + 2b\omega^2$

$f = \sqrt{F_r^2 + F_\theta^2}$